Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Further Pure Mathematics 3R (6669/01R)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2014
Publications Code UA038882

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme		Marks
1.	$5\tanh x + 7 = 5\operatorname{sech} x$		
	$5\dfrac{e^x - e^{-x}}{e^x + e^{-x}} + 7 = \dfrac{10}{e^x + e^{-x}}$	The given equation correctly expressed in terms of exponentials in any form.	B1
	$5\tanh x + 7 = 5\operatorname{sech} x \Rightarrow 5\sinh x + 7\cosh x = 5$ $\Rightarrow 5\dfrac{(e^x - e^{-x})}{2} + 7\dfrac{(e^x + e^{-x})}{2} = 5$	could also score B1	
	$5(e^x - e^{-x}) + 7(e^x + e^{-x}) = 10$		
	$5(e^{2x} - 1) + 7(e^{2x} + 1) = 10e^x$	Attempt quadratic in e^x	M1
	$12e^{2x} - 10e^x + 2 = 0$	Correct quadratic	A1
	$6e^{2x} - 5e^x + 1 = 0 \Rightarrow (3e^x - 1)(2e^x - 1) = 0$	Solves their 3TQ in e^x	M1
	$x = \ln(\tfrac{1}{3}), \quad \ln(\tfrac{1}{2})$	Both correct (Allow –ln3 and/or –ln2)	A1
			(5)
			Total 5
	Alternative 1		
	$5\tanh x + 7 = 5\operatorname{sech} x \Rightarrow 25\tanh^2 x + 70\tanh x + 49 = 25\operatorname{sech}^2 x$		
	$50\tanh^2 x + 70\tanh x + 24 = 0$	Correct quadratic in tanh*x*	B1
	$\tanh x = -\dfrac{4}{5}, \quad \tanh x = -\dfrac{3}{5}$	M1: Solves their 3TQ in tanh*x* A1: Correct values	M1A1
	$\dfrac{e^{2x} - 1}{e^{2x} + 1} = -\dfrac{4}{5} \Rightarrow e^{2x} = \dfrac{1}{9} \Rightarrow x = \ln\dfrac{1}{3}$	M1: Uses the correct exponential form of tanh*x* to obtain a value for *x* at least once	M1A1
	$\dfrac{e^{2x} - 1}{e^{2x} + 1} = -\dfrac{3}{5} \Rightarrow e^{2x} = \dfrac{1}{4} \Rightarrow x = \ln\dfrac{1}{2}$	A1 both answers correct	
	Alternative 2		
	$5\sinh x + 7\cosh x = 5 \Rightarrow 49\cosh^2 x = 25 - 50\sinh x + 25\sinh^2 x$		
	$24\sinh^2 x + 50\sinh x + 24 = 0$	Correct quadratic in sinh*x*	B1
	$\sinh x = -\dfrac{4}{3}, \quad \sinh x = -\dfrac{3}{4}$	M1: Solves their 3TQ in sinh*x* A1: Correct values	M1A1
	$\dfrac{e^x - e^{-x}}{e^x + e^{-x}} = -\dfrac{4}{3} \Rightarrow e^x = \dfrac{1}{3} \Rightarrow x = \ln\dfrac{1}{3}$	M1: Uses the correct exponential form of sinh*x* to obtain a value for *x* at least once	M1A1
	$\dfrac{e^x - e^{-x}}{e^x + e^{-x}} = -\dfrac{3}{4} \Rightarrow e^x = \dfrac{1}{2} \Rightarrow x = \ln\dfrac{1}{2}$	A1 both answers correct	

Question Number	Scheme		Marks
2.(a)	$9x^2+6x+5\equiv a(x+b)^2+c$		
	$a=9,\ b=\frac{1}{3},\ c=4$		B1, B1, B1
			(3)
(b)	$\int\frac{1}{9(x+\frac{1}{3})^2+4}\,dx=\frac{1}{6}\arctan\left(\frac{3x+1}{2}\right)(+c)$	M1: $k\arctan\left(\frac{x+"\frac{1}{3}"}{\sqrt{\frac{"4"}{"9"}}}\right)$ A1: $\frac{1}{6}\arctan\left(\frac{3x+1}{2}\right)$ oe	M1A1
			(2)
(c)	$\int\frac{1}{\sqrt{9(x+\frac{1}{3})^2+4}}\,dx=\frac{1}{3}\text{arsinh}\left(\frac{3x+1}{2}\right)(+c)$	M1: $k\,\text{arsinh}\left(\frac{x+"\frac{1}{3}"}{\sqrt{\frac{"4"}{"9"}}}\right)$ A1: $\frac{1}{3}\text{arsinh}\left(\frac{3x+1}{2}\right)$ oe Allow $\frac{1}{\sqrt{9}}$	M1A1
			(2)
			Total 7

Question Number	Scheme		Marks
	$y = \frac{1}{2}\ln(\coth x)$		
3.(a)	$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\coth x} \times -\text{cosech}^2 x$	M1: Correct use of the chain rule. Allow an expression of the form $\frac{k}{\coth x} \times f(x)$ where f(x) is a hyperbolic function. A1: Correct differentiation	M1A1
	$= \frac{-1}{2\sinh x \cosh x} = \frac{-1}{\sinh 2x} = -\text{cosech}\,2x$*	Completes to printed answer with at least one line of working (e.g as shown) and **no** errors	A1*
(a) Way 2	$e^{2y} = \coth x \Rightarrow 2e^{2y}\frac{dy}{dx} = -\text{cosech}^2 x$	M1: Makes e^y the subject and attempt to differentiate with respect to x A1: Correct differentiation	M1A1
	$\frac{dy}{dx} = \frac{-\text{cosech}^2 x}{2\coth x} = \frac{-1}{\sinh 2x} = -\text{cosech}\,2x$*	Completes to printed answer with no errors	A1*
			(3)
(b)	$S = \int_{(\ln 2)}^{(\ln 3)} (1 + \text{cosech}^2 2x)^{\frac{1}{2}}\, dx$	Substitutes cosech2x into a correct formula (limits not needed)	M1
	$S = \int_{(\ln 2)}^{(\ln 3)} \coth 2x\, dx$	Use of 1 + $\text{cosech}^2 2x$ = $\coth^2 2x$	M1
	$S = \left[\frac{1}{2}\ln(\sinh 2x)\right]_{\ln 2}^{\ln 3}$	Correct integration	A1
	$S = \frac{1}{2}\ln(\sinh(2\ln 3)) - \frac{1}{2}\ln(\sinh(2\ln 2))$	Uses the limits ln2 and ln3 and subtracts either way round. Dependent on first M.	dM1
	$S = \frac{1}{2}\ln\left(\frac{9-\frac{1}{9}}{2}\right)\left(\frac{2}{4-\frac{1}{4}}\right)$	Uses the exponential form of sinh$x$ **and** combines ln's to give an expression in terms of ln only. **Dependent on the first and 3rd M.**	ddM1
	$S = \frac{1}{2}\ln\frac{64}{27}$ or $\frac{3}{2}\ln\frac{4}{3}$		A1
			(6)
			Total 9

Question Number	Scheme		Marks
4.	$\int_0^{\sqrt{3}} (3-x^2)^n \mathrm{d}x$		
(a)	$\int_0^{\sqrt{3}} (3-x^2)^n \mathrm{d}x = \left[x(3-x^2)^n\right]_0^{\sqrt{3}} + \int_0^{\sqrt{3}} 2x^2 n(3-x^2)^{n-1}\mathrm{d}x$		M1A1
	M1: Integration by parts in the correct direction A1: Correct expression (Ignore limits)		
	$= 0 - 2n\int_0^{\sqrt{3}} (3-x^2-3)(3-x^2)^{n-1}\mathrm{d}x$	Substitutes limits and uses $x^2 = x^2 - 3 + 3$ **Dependent on the first M**	**d**M1
	$= 0 - 2n\int_0^{\sqrt{3}} (3-x^2)^n \mathrm{d}x + 6n\int_0^{\sqrt{3}} (3-x^2)^{n-1}\mathrm{d}x$	Correct expressions	A1
	$= 6nI_{n-1} - 2nI_n$	Substitutes for I_{n-1} and I_n **Dependent on both M's**	**dd**M1
	$I_n = \frac{6n}{2n+1} I_{n-1}$ *	Correct completion with **no** errors	A1*
			(6)
	(a) Alternative		
	$I_n = \int_0^{\sqrt{3}} (3-x^2)^{n-1}\left(3-x^2\right)\mathrm{d}x = 3\int_0^{\sqrt{3}} (3-x^2)^{n-1}\mathrm{d}x - \int_0^{\sqrt{3}} x^2(3-x^2)^{n-1}\mathrm{d}x$		dM1
	M1: Writes the bracket as a product and separates into two integrals **This is the second M and depends on the first M below**		
	$= 3I_{n-1} - \int_0^{\sqrt{3}} x \times x(3-x^2)^{n-1}\mathrm{d}x$		
	$= 3I_{n-1} - \left\{\left[\frac{x\left(3-x^2\right)^n}{-2n}\right]_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{\left(3-x^2\right)^n}{-2n}\mathrm{d}x\right\}$	M1: Parts in the correct direction (First M1) A1: Correct expression (First A1)	M1A1
	$= 3I_{n-1} - \int_0^{\sqrt{3}} \frac{\left(3-x^2\right)^n}{2n}\ \mathrm{d}x$	Correct expression with no errors	A1
	$= 3I_{n-1} - \frac{1}{2n} I_{n-1}$	Substitutes for I_{n-1} and I_n **Dependent on both M's**	**dd**M1
	$I_n = \frac{6n}{2n+1} I_{n-1}$ *	Correct completion with no errors	A1
(b)	$I_0 = \sqrt{3}$ *or* $I_1 = 2\sqrt{3}$		B1
	$I_4 = \frac{24}{9} I_3$	Attempt I_4 in terms of I_3	M1
	$I_4 = \frac{24}{9}.\frac{18}{7}I_2 = \frac{24}{9}.\frac{18}{7}.\frac{12}{5}I_1$	M1: Attempt I_4 in terms of I_1 A1: Correct expression for I_4 as shown or correct numerical expression	M1A1
	$I_4 = \frac{24}{9}.\frac{18}{7}.\frac{12}{5}.\frac{6}{3}.\sqrt{3}$		
	$I_4 = \frac{1152}{35}\sqrt{3}$		A1
			(5)
			Total 11

Question Number	Scheme		Marks
5.(a)	$a = 3,\ b = 1$	Both	B1
			(1)
(b)	$\frac{dy}{dx} = -\frac{\cos\theta}{3\sin\theta}$	Complete correct gradient method including use of coordinates	M1
	$y - \sin\theta = -\frac{\cos\theta}{3\sin\theta}(x - 3\cos\theta)$ (I)	Correct straight line method	M1
	$3y\sin\theta - 3\sin^2\theta = -x\cos\theta + 3\cos^2\theta$		
	Allow both M's if working in *a* and *b* so far		
	$3y\sin\theta + x\cos\theta = 3\cos^2\theta + 3\sin^2\theta = 3*$	Correct completion to printed answer with no errors seen. Some working is needed from (I) to *.	A1*
			(3)
(c)	$x = 0 \Rightarrow y = \frac{1}{\sin\theta},\ y = 0 \Rightarrow x = \frac{3}{\cos\theta}$	Both	B1
	$\text{Area} = \frac{1}{2} \times "\frac{1}{\sin\theta}" \times "\frac{3}{\cos\theta}"$	Correct method for area	M1
	$= \frac{3}{2\sin\theta\cos\theta} = 3\text{cosec}2\theta$		A1
			(3)
(d)	$x = \frac{3}{2\cos\theta},\ y = \frac{1}{2\sin\theta}$	Correct follow through mid-point	B1ft
	$\sin\theta = \frac{1}{2y},\ \cos\theta = \frac{3}{2x}$ $\left(\frac{3}{2x}\right)^2 + \left(\frac{1}{2y}\right)^2 = 1$	Attempt sin and cos in terms of x and y and attempt Pythagoras. Allow if x and y are exchanged.	M1
	$9y^2 + x^2 = 4x^2y^2$		
	$y^2(4x^2 - 9) = x^2 \Rightarrow y^2 = ...$	Attempt to isolate y^2	M1
	$y^2 = \frac{x^2}{4x^2 - 9}$	Correct equation (oe)	A1
			(4)
(d) Way 2	$x = \frac{3}{2\cos\theta},\ y = \frac{1}{2\sin\theta}$	Correct follow through mid-point	B1ft
	$y^2 = \frac{1}{4\sin^2\theta}$	Attempt y^2 in terms of sin	M1
	$y^2 = \frac{1}{4\left(1 - \cos^2\theta\right)}$	Correct use of Pythagoras	M1
	$y^2 = \frac{1}{4\left(1 - \frac{9}{4x^2}\right)}$	Correct equation (oe)	A1
			Total 11

Question Number	Scheme		Marks
6.(a)	$\mathbf{P} = \begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix} \left(= \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{pmatrix} \right)$	M1: Attempt unit eigenvectors A1: Correct matrix	M1A1
	$\mathbf{D} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	M1: Correct form for **D** with eigenvalues in the diagonal A1: Consistent with **P**	M1A1
			(4)
(b)	$\mathbf{MP} = \mathbf{PD}$ or $\mathbf{P}^{-1}\mathbf{M} = \mathbf{DP}^{-1}$		M1
	$\mathbf{M} = \mathbf{PDP}^{-1}$		A1
			(2)
(c)	$\mathbf{P}^{-1} = \mathbf{P}^{\mathrm{T}} = \frac{1}{3}\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix}$	Correct matrix. Allow the transpose of their **P**.	B1ft
	$\mathbf{PD} = \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 10 & -4 & -1 \\ 10 & 2 & 2 \\ 5 & 4 & -2 \end{pmatrix}$ **or** $\mathbf{DP}^{-1} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}\frac{1}{3}\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 10 & 10 & 5 \\ -4 & 2 & 4 \\ -1 & 2 & -2 \end{pmatrix}$		M1A1
	M1: Attempt **PD** or $\mathbf{DP}^{-1}$ where $\mathbf{D} \neq k\mathbf{I}$ A1: Correct matrix		
	$\mathbf{M} = \frac{1}{9}\begin{pmatrix} 10 & -4 & -1 \\ 10 & 2 & 2 \\ 5 & 4 & -2 \end{pmatrix}\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{pmatrix}$ **Or** $\mathbf{M} = \frac{1}{9}\begin{pmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 10 & 10 & 5 \\ -4 & 2 & 4 \\ -1 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{pmatrix}$		M1A1
	M1: Completes correctly to find **M** A1: Correct **M**		
	Failure to use unit eigenvectors in (a) could score M0A0M1A1 in (a) and B1ftM1A0M1A0 in (c)		
			(5)
			Total 11

Question Number	Scheme		Marks
7.(a)	$y = e^{-x} \Rightarrow \frac{dy}{dx} = -e^{-x}$	Correct derivative	B1
	$S = 2\pi\int y\sqrt{1+(y')^2}\,dx = 2\pi\int e^{-x}\sqrt{1+e^{-2x}}\,dx$	M1: Use of correct formula A1: Correct proof with no errors	M1A1
			(3)
(b)	$e^{-x} = \sinh u \Rightarrow -e^{-x} = \cosh u\frac{du}{dx}$	Correct differentiation	B1
	$S = 2\pi\int e^{-x}\sqrt{1+e^{-2x}}\,dx = 2\pi\int \sinh u\sqrt{1+\sinh^2 u}.\frac{\cosh u}{-\sinh u}\,du$		M1
	A complete substitution		
	$(2\pi)\int \sinh u\sqrt{1+\sinh^2 u}.\frac{\cosh u}{-\sinh u}\,du = -2\pi\int \cosh^2 u\,du$		A1
	$x = 0 \Rightarrow u = \text{arsinh}(1)(= \ln(1+\sqrt{2}))$ $x = \ln 3 \Rightarrow u = \text{arsinh}(\frac{1}{3})(= \ln(\frac{1}{3}+\sqrt{1+\frac{1}{9}}))$	Both limits correct	B1
	$S = 2\pi\int_{\text{arsinh}(\frac{1}{3})}^{\text{arsinh}(1)} \cosh^2 u\,du$	Correct completion with **no** errors	A1
			(5)
(c)	$2\int \cosh^2 u\,du = \int(\cosh 2u + 1)\,du$	Uses $2\cosh^2 u = \pm\cosh 2u \pm 1$	M1
	$= \frac{1}{2}\sinh 2u + u(+k)*$	cso	A1*
			(2)
(d)	$S = \pi(\frac{1}{2}\sinh 2(\text{arsinh}\beta) + \text{arsinh}\beta - \frac{1}{2}\sinh 2(\text{arsinh}\alpha) - \text{arsinh}\alpha)$		M1
	Attempt to use their limits (subtracting either way round) (allow the omission of π and allow 2 π instead of π) There must be some evidence of the use of their limits e.g. an answer of 5.08 with no working loses this mark		
	= 5.079	Cao (Allow recovery from -5.079)	A1
			(2)
			Total 12
	NB $S = \pi(\sqrt{2} + \ln(1+\sqrt{2}) - \frac{1}{3}\frac{\sqrt{10}}{3} - \ln(\frac{1}{3}+\frac{\sqrt{10}}{3})) = 5.079241597$		

Question Number	Scheme		Marks
8.(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 3 \\ -1 & 2 & 4 \end{vmatrix} = \begin{pmatrix} -2 \\ -11 \\ 5 \end{pmatrix}$	M1: Attempt cross product of normal vectors. If method unclear, 2 components must be correct. A1: Correct vector	M1A1
	$x=0:(0, \frac{1}{2}, \frac{3}{2}),\quad y=0:(-\frac{1}{11}, 0, \frac{19}{11}),\quad z=0:(\frac{3}{5}, \frac{19}{5}, 0)$		M1A1
	M1: Attempt point on the line (x, y and z). A1: Correct coordinates		
	$\mathbf{r} = \frac{1}{2}\mathbf{j} + \frac{3}{2}\mathbf{k} + \lambda(2\mathbf{i} + 11\mathbf{j} - 5\mathbf{k})$	M1: Their point + λ their direction **Dependent on both previous method marks.** A: Correct equation (oe)	**dd**M1A1
			(6)
	Alternative 1		
	$x = \frac{y-\frac{1}{2}}{\frac{11}{2}} = \frac{z-\frac{3}{2}}{-\frac{5}{2}}$ or $\frac{x+\frac{1}{11}}{\frac{2}{11}} = y = \frac{z-\frac{19}{11}}{-\frac{5}{11}}$ or $\frac{x-\frac{3}{5}}{-\frac{2}{5}} = \frac{y-\frac{19}{5}}{-\frac{11}{5}} = z$		M1A1
	M1: Correctly attempts cartesian equations of line A1: Correct equations		
	$(0, \frac{1}{2}, \frac{3}{2})$ or $(-\frac{1}{11}, 0, \frac{19}{11})$ or $(\frac{3}{5}, \frac{19}{5}, 0)$ M1: Extracts position correctly A1: Correct position **or** $\lambda(2\mathbf{i} + 11\mathbf{j} - 5\mathbf{k})$ M1: Extracts direction correctly A1: Correct direction		M1A1
	$\mathbf{r} = \frac{1}{2}\mathbf{j} + \frac{3}{2}\mathbf{k} + \lambda(2\mathbf{i} + 11\mathbf{j} - 5\mathbf{k})$	M1: Their point + λ their direction **Dependent on both previous method marks.** A: Correct equation (oe)	**dd**M1A1
	Alternative 2		
	$x = \lambda \Rightarrow y = \frac{11\lambda+1}{2},\ z = \frac{3-5\lambda}{2}$ (oe)	M1:Obtains x, y and z in terms of "λ" A1: Correct expressions	M1A1
	$(0, \frac{1}{2}, \frac{3}{2})$ **or** $\lambda(2\mathbf{i} + 11\mathbf{j} - 5\mathbf{k})$	M1: Extracts position correctly A1: Correct position or M1: Extracts direction correctly A1: Correct direction	M1A1
	$\mathbf{r} = \frac{1}{2}\mathbf{j} + \frac{3}{2}\mathbf{k} + \lambda(2\mathbf{i} + 11\mathbf{j} - 5\mathbf{k})$	M1: Their point + λ their direction **Dependent on both previous method marks.** A: Correct equation (oe)	**dd**M1A1
(b)	$2\lambda - \left(\frac{1}{2} + 11\lambda\right) + 2\left(\frac{3}{2} - 5\lambda\right) = 31$	Substitutes into the third plane and solves for λ	M1
	$\lambda = \frac{-3}{2}$		
	Planes intersect at (-3, -16, 9)	M1: Substitutes into their line A1: Correct coordinates	M1A1
			(3)
	Alternative		
	M1: Solves three simultaneous equations to obtain one value for x or y or x M1: Solves three simultaneous equations to obtain values for x, y and x A1: Correct coordinates		
			Total 9